

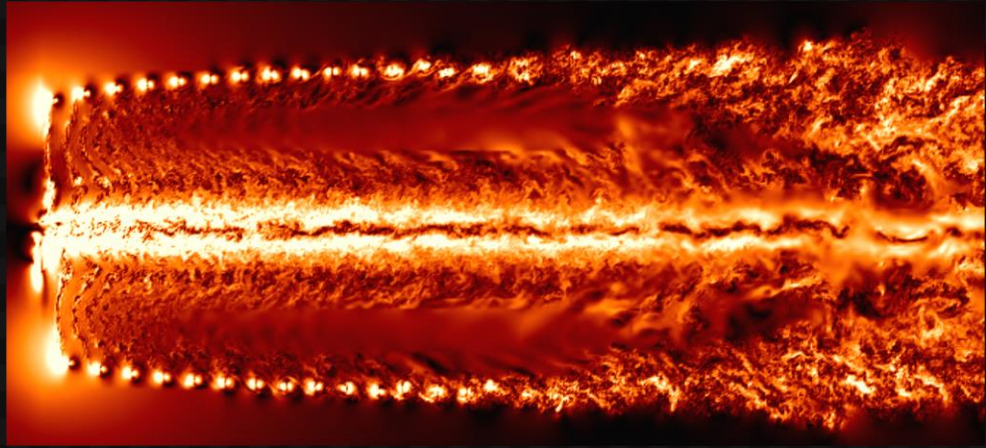
IEEE HPEC 1-1: General Purpose GPU Computing Session

# GPU-Accelerated Discontinuous Galerkin Methods: 30x Speedup on 345 Billion Unknowns

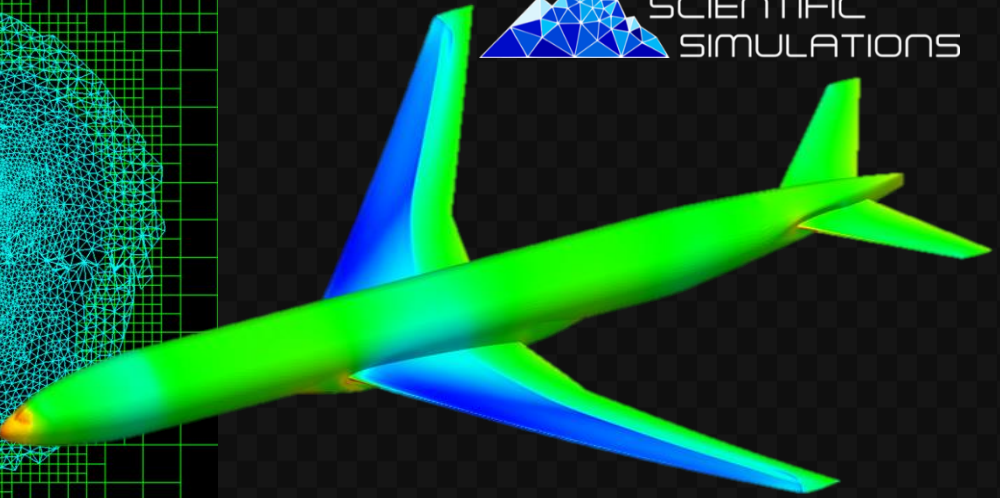
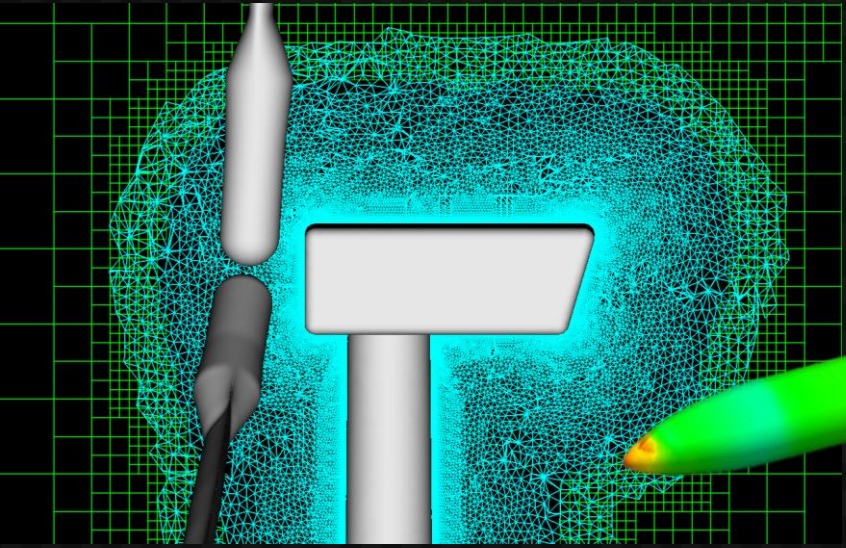
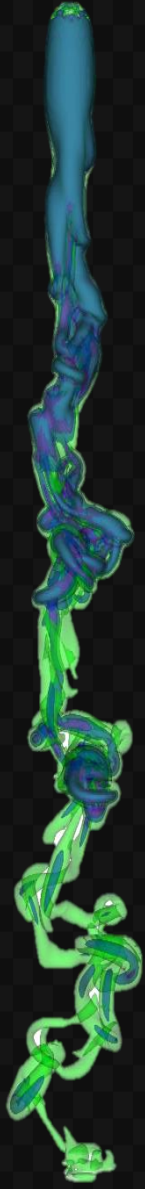
September 21, 2020



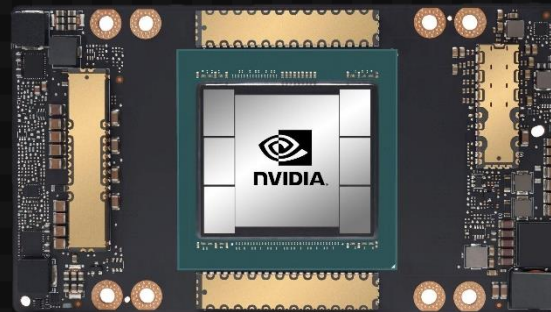
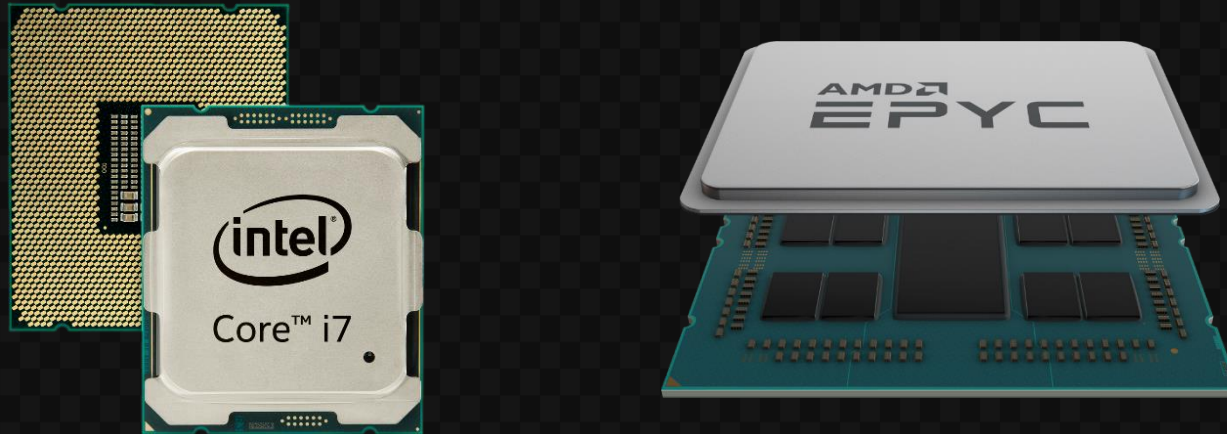
Andrew C. Kirby<sup>1</sup>  
Dimitri Mavriplis<sup>2</sup>



Wyoming Wind and Aerospace Applications Komputation Environment



- Leadership-Class Supercomputing Facilities: Heterogeneous Computing



NVIDIA A100  
9.7 TFLOPS Double Precision



## Compressible Euler Equations Cartesian Meshes

$$\frac{\partial \mathbf{Q}(\mathbf{x}, t)}{\partial t} + \vec{\nabla} \cdot \mathbf{F}(\mathbf{Q}(\mathbf{x}, t)) = 0$$

$$\mathbf{Q} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{Bmatrix}, \mathbf{F} = \begin{Bmatrix} \underline{\mathbf{F}}^1 & \underline{\mathbf{F}}^2 & \underline{\mathbf{F}}^3 \\ \rho u & \rho v & \rho w \\ \rho u^2 + p - \tau_{11} & \rho uv - \tau_{12} & \rho uw - \tau_{13} \\ \rho uv - \tau_{21} & \rho v^2 + p - \tau_{22} & \rho vw - \tau_{23} \\ \rho uw - \tau_{31} & \rho vw - \tau_{32} & \rho w^2 + p - \tau_{33} \\ \rho uH + q_1 - \tau_{1j}u_j & \rho vH + q_2 - \tau_{2j}u_j & \rho wH + q_3 - \tau_{3j}u_j \end{Bmatrix}$$

$$\frac{\partial \mathbf{Q}(\mathbf{x}, t)}{\partial t} + \vec{\nabla} \cdot \mathbf{F}(\mathbf{Q}(\mathbf{x}, t)) = 0$$

Discontinuous Galerkin  
Method

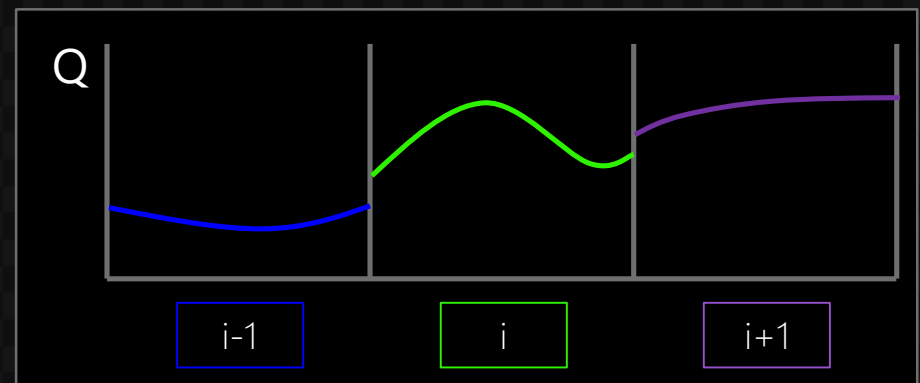
$$\int_{\Omega_k} \left( \frac{\partial \mathbf{Q}}{\partial t} + \vec{\nabla} \cdot \mathbf{F} \right) \psi(\mathbf{x}) d\mathbf{x} = 0$$

$$\mathbf{R}^{\text{Weak}} = \int_{\Omega_k} \frac{\partial \mathbf{Q}}{\partial t} \psi(\mathbf{x}) d\mathbf{x} - \int_{\Omega_k} (\mathbf{F} \cdot \vec{\nabla}) \psi(\mathbf{x}) d\mathbf{x} + \int_{\Gamma_k} (\mathbf{F}^* \cdot \vec{\mathbf{n}}) \psi(\mathbf{x}|_{\Gamma_k}) d\Gamma_k = 0$$

## Nodal Basis Functions

$$l_s(x) = \prod_{i=1, i \neq s}^N \frac{(x - \xi_i)}{(\xi_s - \xi_i)}$$

Lagrange Interpolating Polynomial  
One-Dimensional

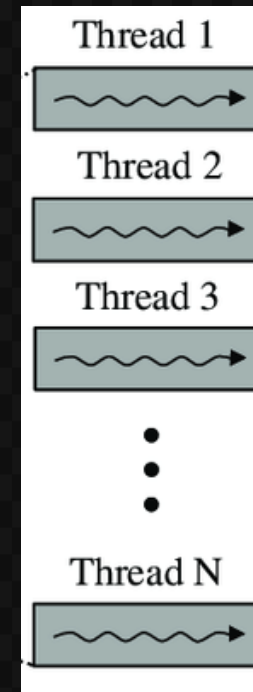
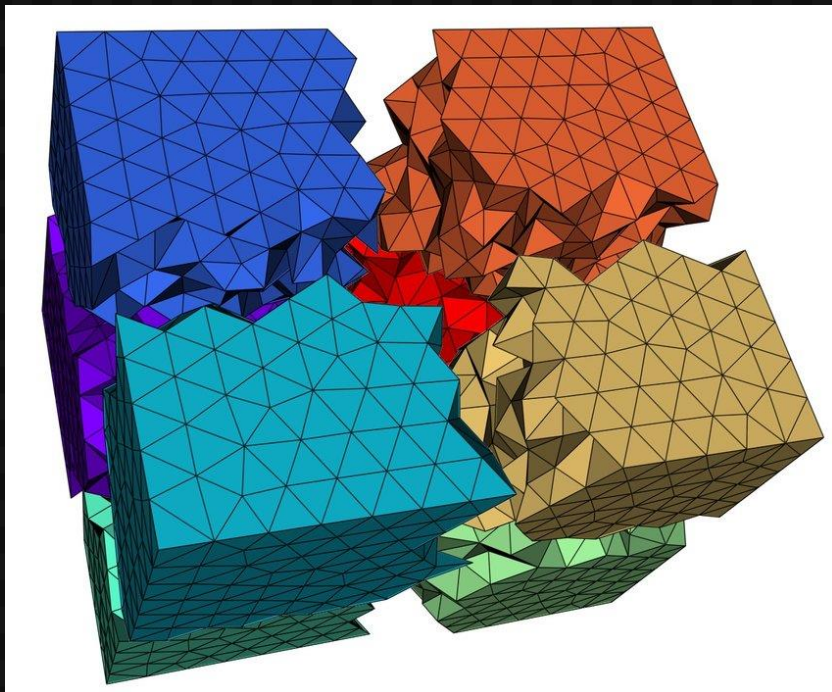


## MPI + X Programming Model

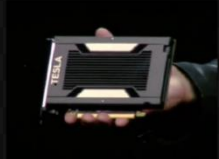
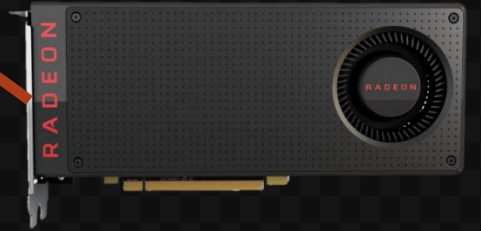
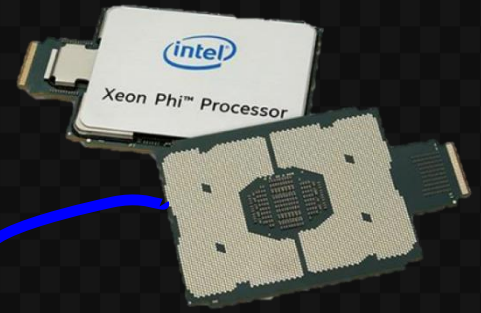
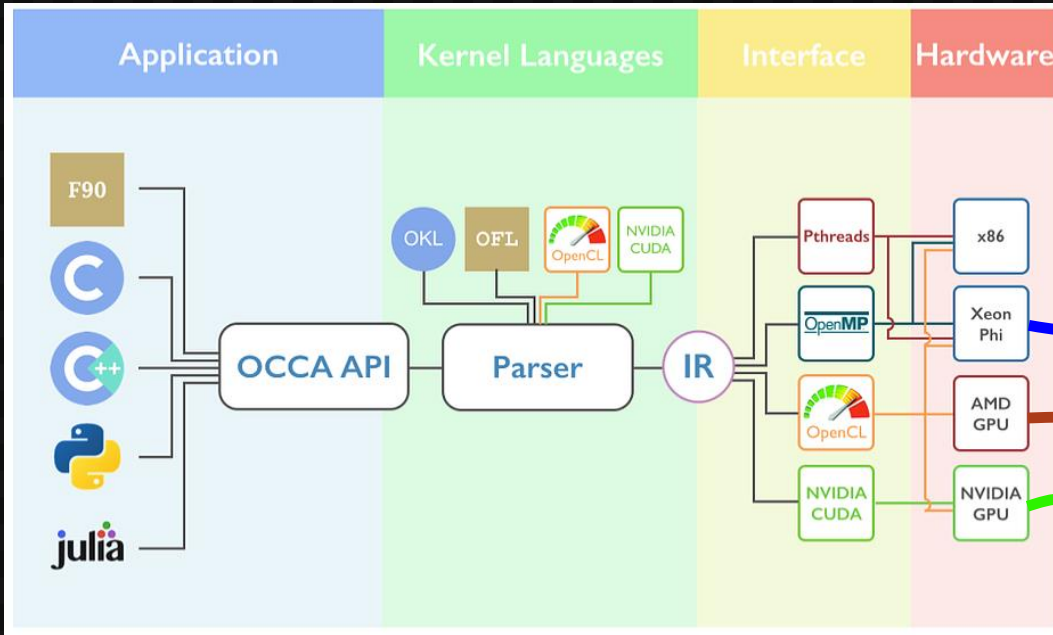
$$\mathbb{M} \frac{\partial \mathbf{Q}_{ijk}(t)}{\partial t} + \mathbf{R}_{ijk}(\mathbf{Q}) = 0$$

MPI: coarse-grained parallelism

X: fine-grained parallelism



A Unified API for programming devices



# MIT Satori Results

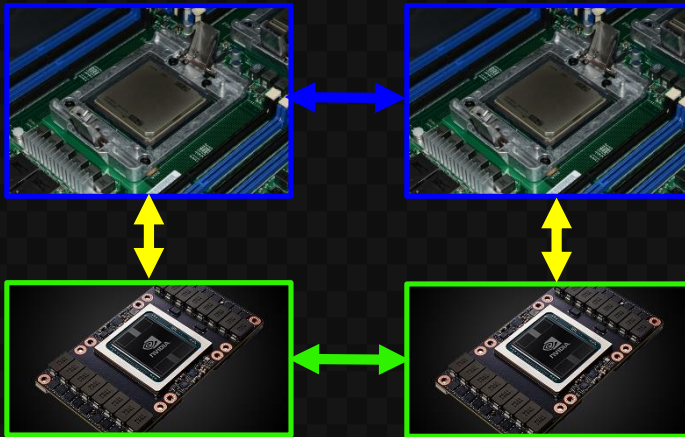


## 64 Nodes

- 2 IBM Power9 CPUs
- 4 NVIDIA Volta V100
- EDR Infiniband

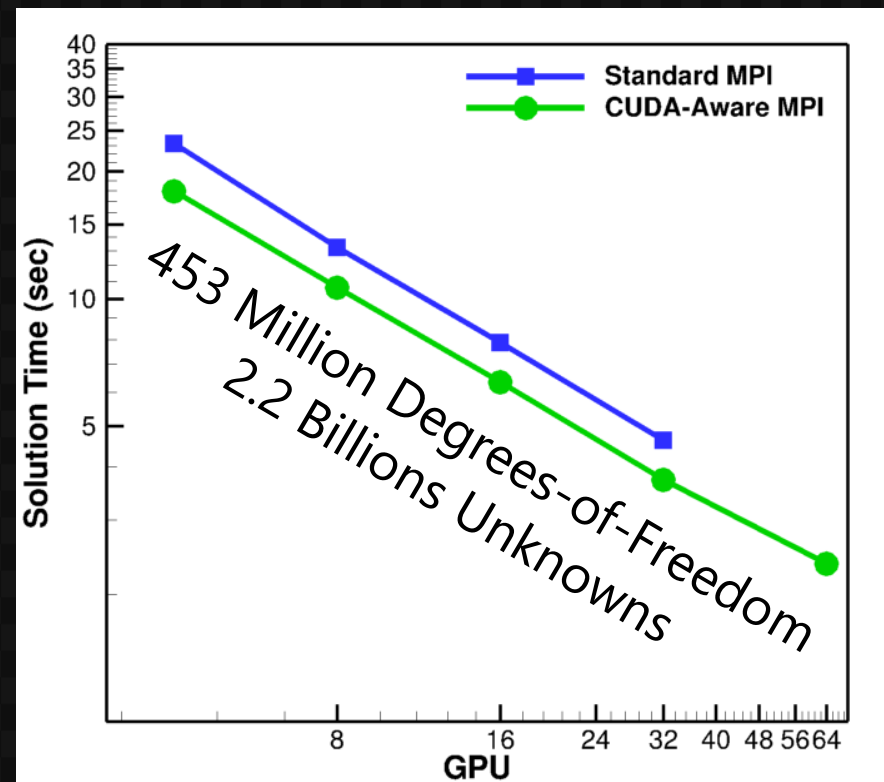
## Total

- 256 GPUs
- #7 Green500



**MIT Satori: CUDA-Aware MPI**  
Problem Size: 452,984,832 DOF.

Solution Time (sec)			
GPUs	ON	OFF	Speedup
4	17.94	23.22	1.29x
8	10.60	13.22	1.25x
16	6.34	7.87	1.24x
32	3.73	4.62	1.24x





## 4,608 Nodes

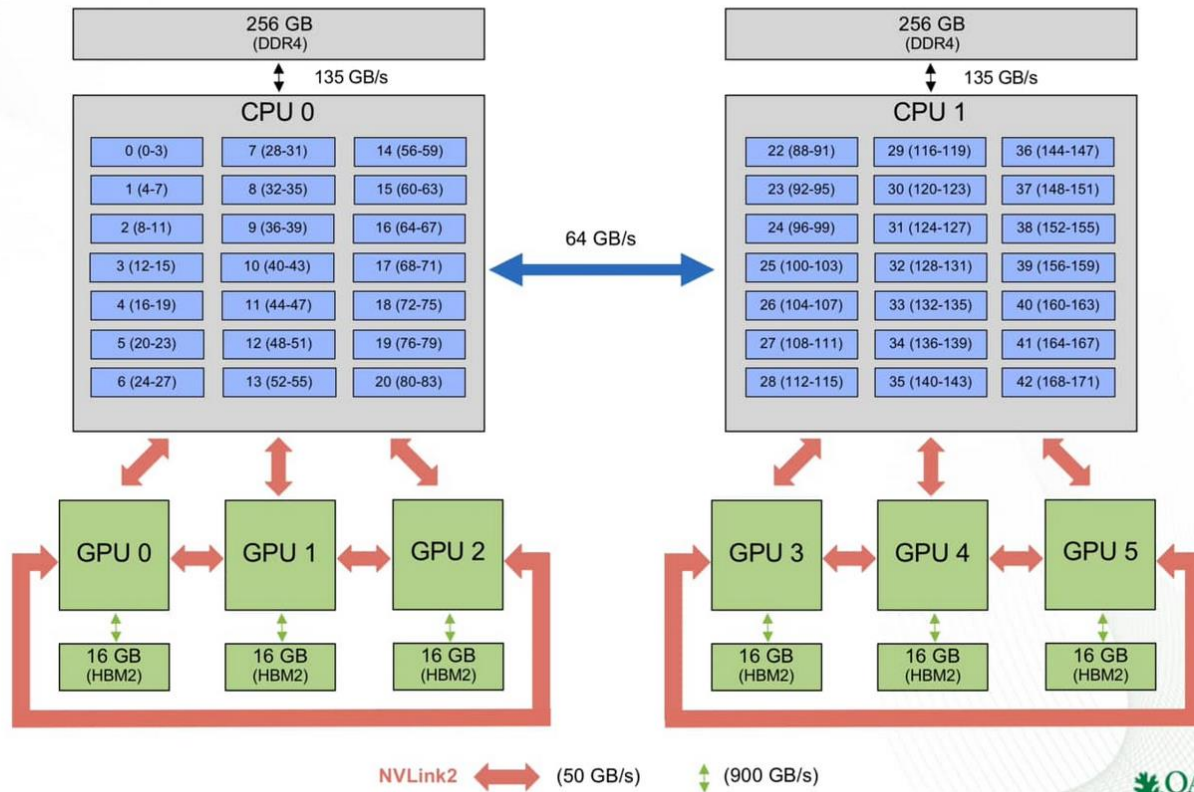
- 2 IBM Power9 CPUs
- 6 NVIDIA Volta V100

## Total

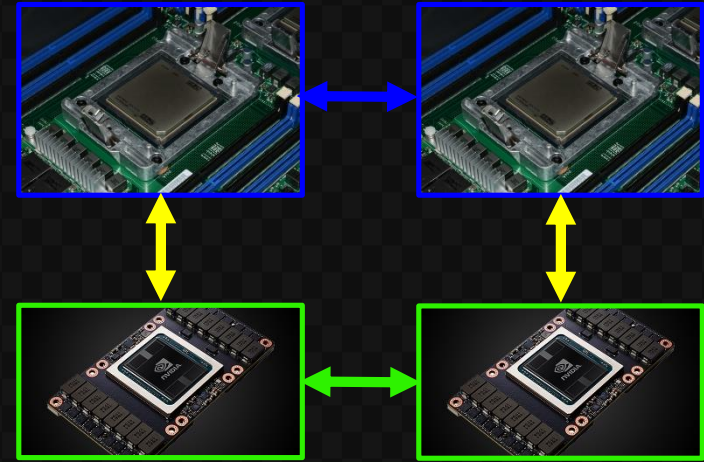
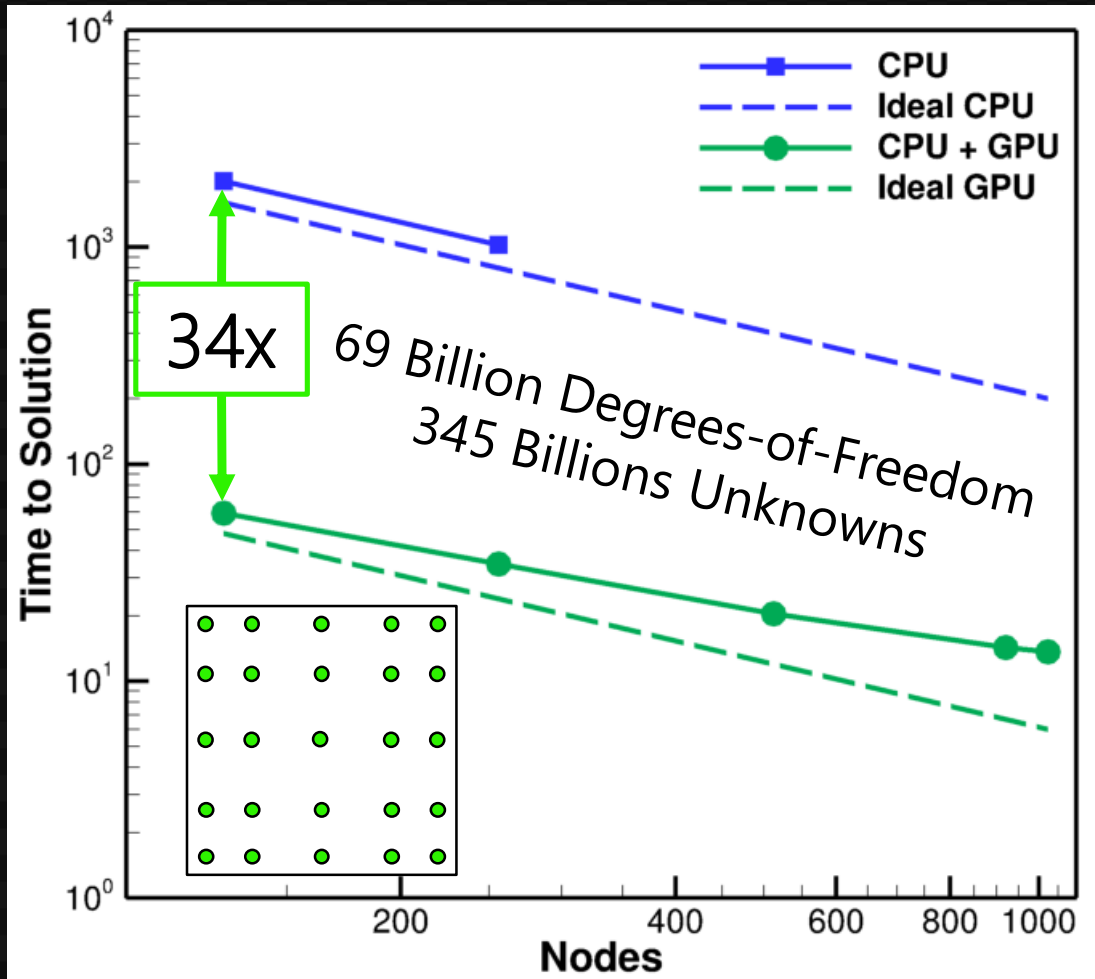
- 27,648 GPUs
- 200 PFLOPs



### Summit Node (2) IBM Power9 + (6) NVIDIA Volta V100



## Non-CUDA-Aware MPI



30% Speedup with CUDA-Aware MPI

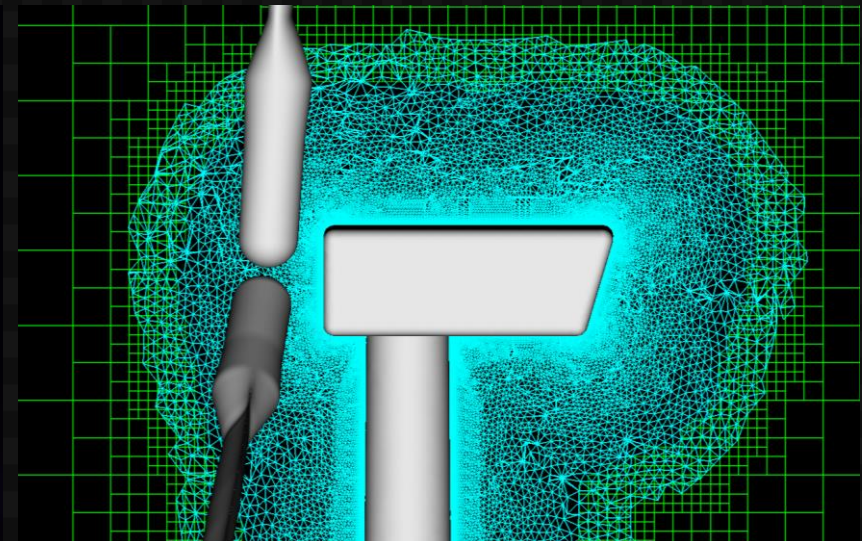
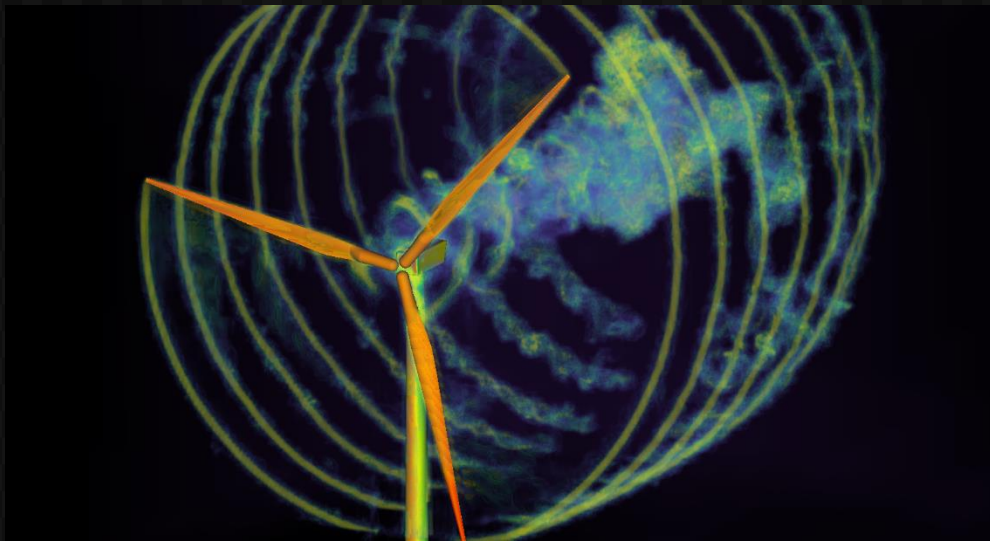
CUDA-Aware MPI		
Kernel	Time (sec)	
	ON	OFF
Volume	1.67	1.68
Surface	2.04	2.04
Update-Project	5.70	5.70
Communication	0.65	5.13
Overall	10.37	14.86

## Results

- Demonstrated good performance on GPUs  
~30x speedup
- Demonstrated 24-30% improvement using GPU-Direct MPI

## Future Work

- Extend GPU implementation to Adaptive Mesh Refinement on unstructured hex/quad meshes
- Extend GPU implementation to overset mesh capabilities
- 1 Trillion DOF Simulations



# Acknowledgements

## Compute Time

MIT: Satori Supercomputer

ORNL: Summit Supercomputer

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## U.S. Department of Energy, Office of Science, Basic Energy Sciences

Award #DE-SC0012671



Thank You  
Questions?

